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Continuously Dislocated Elastic Bodies with a Neo-Hookean Like Energy Subjected to Anti-plane Shear

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Abstract In this paper, we consider a materially uniform but inhomogeneous body and we are interested in three particular cases of inhomogeneities corresponding to three distinct distributions of dislocations. The field of defects enters the equilibrium equations through the components of the tensor field describing the relaxation procedure. We examine what form should these components take in order for the material to admit states of anti-plane shear. The results obtained in this paper hold for a class of materials that obey a specific form for the stored energy function. In the special case of no dislocations, this class falls under the well known class of Neo-Hookean materials.

Keywords Anti-pane shear · Materially uniform · Inhomogeneous bodies

Mathematics Subject Classification (2000) 74A05 · 74A99 · 74C99

1 Introduction

Deformations of the anti-plane shear type are characterized by the fact that the elastic field is fully described by a scalar function. This scalar field is called the "out-of-plane displacement" of anti-plane shear and knowledge of this quantity is equivalent to knowledge of the elastic field. Adkins [5] seems to have been the first who studied deformations belonging to this class for an incompressible isotropic elastic body (see also Green and Adkins [8] and

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references therein). Knowles [10] extended Adkins' work by providing the general form that the stored energy should have in order that an isotropic and incompressible material admits states of anti-plane shear. He also used the framework of anti-plane shear in a crack problem [11]. In the same paper he showed how the three differential equations of equilibrium can be reduced to a single one for the case of generalized neo-Hookean materials. The same author provided a necessary and sufficient condition for the strain energy density in order that a compressible isotropic material admits states of anti-plane shear [12]. Jiang and Knowles [9] determined a special class of a compressible isotropic body that can sustain anti-plane shear. Tsai and Rosakis [25], working on anisotropic bodies in the dynamical case, provided a necessary and sufficient condition that the stored energy function should obey, for a compressible material to admit states of anti-plane shear.

Since, for the case of anti-plane shear, the elastic field reduces to a scalar function, the field equations constitute, in general, an over-determined system for incompressible bodies. The two unknown functions—the pressure field, p and the out-of-plane displacement, u—should satisfy three differential equations. In general, not every pair of solutions for the two unknown functions (u, p), that satisfy the two differential equations of equilibrium, will also satisfy the third. If this happens we can say that the equations are consistent. Also, in the case where there exists an expression relating the pressure field with the out-of-plane displacement in such a way that two of the differential equations are satisfied identically, then only one equation for the determination of u remains. The pressure field will be found by substituting the solutions for u in the equation associating the pressure field with u.

The inconsistency that in general arises, allows one to expect that not every material is capable of sustaining deformations of this kind, unless its response to applied loads, which is measured through the stored energy function, is restricted somehow. This restriction is given in the form of a necessary and sufficient condition, for the stored energy function, in the works of the previously mentioned authors for some classes of materials. We should note that in the framework adopted in this paper, we do not search for expressions that the energy functions should obey in order for the material to admit states of anti-plane shear, namely, the equations of equilibrium to be consistent. A specific expression for the energy function is used. This expression reduces to that of a neo-Hookean elastic body when the defects are absent.

For the case of a materially uniform but inhomogeneous body [18], where the inhomogeneities are due to a fixed continuous distribution of dislocations, the situation is slightly changed. The dislocations field enters the field equations, since the presence of the defects alters the constitutive expression of the stresses [24, 26]. The tensor field describing the transformation from the dislocated reference into the stress-free uniform reference configuration [6, 23] characterize the field of dislocations in the elastic body. The components of this tensor field enters the equilibrium equations which deviate from its standard form. Although we treat these quantities as known, we investigate what form they should take in order for the material to admit states of anti-plane shear. Namely, under what conditions for these quantities will there exist a pressure field, such, that the three differential equations of equilibrium can be consistently reduced in a single differential equation for u.

In essence, we search for an expression that relates the pressure field with the out-of-plane displacement and the dislocations field, in such a way that the two differential equations of the fields are satisfied identically. The remaining equation of equilibrium is then the only equation that the out-of-plane displacement should satisfy. This procedure is accomplished in two particular cases of continuous distribution of dislocations—one of the edge and one of the screw type in Sects. 3.1 and 3.2, respectively. For a different distribution of edge dislocations in Sect. 3.3 we give a particular solution for the differential equations of equilibrium under certain conditions.

The analysis of the present work is restricted to bodies subjected to anti-plane deformation possessing three particular types of frozen dislocations and a specific expression for the energy. For a more general setting, one may look at the literature where there are enough works accounting for dislocation motion. For instance, Acharya [1–3] in a series of recent papers has proposed a field theory for a body containing a continuous distribution of dislocations. After having healed a non-uniqueness that arises in the elastic field when the dislocation density tensor is known, he stated a closed set of governing equations for the general dynamical problem. Numerical formulations and solutions within the context of the linear theory are presented by Roy and Acharya [19]. An approach with a link between the continuum theory of dislocations and the continuum plasticity theory has also been presented by the same authors [4, 20]. Earlier attempts on building a dynamical field theory of continuously dislocated bodies can be found in the works of Willis [27], Kosevich [13], Kroner [14] and Mura [17]. In all these approaches the dislocation density tensor is used as the basic field describing the defects.

On the other hand, one can take the view that the plastic part of the deformation gradient, in the language of multiplicative decomposition, is the primarily field in a continuous theory of dislocations. In that case, it is apparent that additional equations are needed for this field. Sedov and Berdichevsky [21], as well as Le and Stumpf [15] produced such equations based on variational principles. For the same purpose, Maugin [16] used a principle of virtual work. In our approach on the subject an appropriate invariance for an augmented balance of energy has been used [22].

The article is organized as follows. Some prerequisites concerning kinematics, constitutive relations and other useful notions are described in the section that follows. In Sect. 3 the main results of our approach are given. The article ends with some concluding remarks in Sect. 4.

2 Basic Considerations

As usual in anti-plane shear, our body is assumed to be an infinite cylinder with generators parallel to X_3 direction. For the kinematics, a third configuration is needed apart from the standard two that elasticity uses [6, 18, 23]. To obtain this additional configuration the dislocated reference B_R is assumed to relax from the internal stresses due to the dislocations presence. The mapping that describes this transformation is denoted by $\mathbf{K}^{-1}$. The latter tensor field describes inhomogeneities that arise due to a fixed distribution of dislocations. The local stress-free configuration into which B_R is transformed under $\mathbf{K}^{-1}$ is called the uniform reference configuration and denoted by B_U. The transformation from B_R to the current configuration B_C is accomplished by the deformation gradient $\mathbf{F}$. Latin letters are used for quantities in B_R and B_C. Upper case for B_R and lower case for B_C. Lower case Greek letters are used for quantities in B_U.

Note that the tensor fields $\mathbf{K}^{-1}$ and $\mathbf{F}$ are considered as independent to each other. The tensor field that describes the transformation from B_U to B_C is denoted by $\mathbf{M}$ and is thought of as being the dependent variable. It is defined to be the composition of the maps $\mathbf{K}^{-1}$ and $\mathbf{F}$ as follows $\mathbf{M} = \mathbf{FK}$. Taking the viewpoint of the multiplicative decomposition of elastoplasticity, one can say that $\mathbf{K}^{-1}$ and $\mathbf{M}$ corresponds to the plastic $\mathbf{F}^{pl}$ and elastic $\mathbf{F}^{el}$ part of $\mathbf{F}$, respectively. We should also mention that since the distribution of dislocations is not allowed to vary, we remain in the elastic regime.

Consider an anti-plane shear deformation field of the form

$$\begin{aligned} x_1 &= X_1, \\ x_2 &= X_2, \\ x_3 &= X_3 + u(X_1, X_2), \end{aligned} \tag{2.1}$$

where the scalar field $u = u(X_1, X_2)$ is the out-of-plane displacement. Then, the deformation gradient takes the following matrix form

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ u,_1 & u,_2 & 1 \end{pmatrix}. \tag{2.2}$$

The comma denotes partial derivative with respect to the material index that follows.

Following standard invariance arguments, the stored energy function per unit volume in B_R can be expressed as

$$w = \bar{W}(\mathbf{F}, \mathbf{X}) = W(\mathbf{C}, \mathbf{X}),$$

where $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ is the right Cauchy-Green deformation tensor. We assume that there exist functions W_U and $\bar{W}_U$ such that

$$W(\mathbf{C}, \mathbf{X}) = W_U(\mathbf{M}^T\mathbf{M}), \qquad \bar{W}(\mathbf{F}, \mathbf{X}) = \bar{W}_U(\mathbf{M}). \tag{2.3}$$

Therefore, the body is considered as materially uniform in the sense of Noll [18]. After the above assumption, the second and first Piola-Kirchhoff stress tensor take the form:

$$\mathbf{S} = 2\frac{\partial W}{\partial \mathbf{C}} = 2\mathbf{K}\frac{\partial W_U(\mathbf{M}^T\mathbf{M})}{\partial(\mathbf{M}^T\mathbf{M})}\mathbf{K}^T, \tag{2.4}$$

$$\mathbf{P} = \frac{\partial \bar{W}}{\partial \mathbf{F}} = \frac{\partial \bar{W}_U}{\partial \mathbf{M}}\mathbf{K}^T, \tag{2.5}$$

respectively.

We consider now the following constitutive assumption for the energy

$$W_U = \frac{\mu}{2}\left[\mathrm{tr}(\mathbf{M}^T\mathbf{M}) - 3\right], \tag{2.6}$$

where μ ($\neq 0$) is the shear modulus. Notice that the above expression for the energy is reduced to the one used for neo-Hookean materials when dislocations are absent, namely, when $\mathbf{K}^{-1} = \mathbf{I}$. Also, a pressure field will appear in the constitutive relation for the stress to compensate for the incompressibility constraint. This scalar field, denoted by $p = p(X)$, is not constitutively determined since it does not, in general, participate in local forms of the second thermodynamic law. It should be determined by the equilibrium conditions.

Introducing (2.6) into (2.4) and (2.5) we obtain [26]

$$\mathbf{P} = \mu\mathbf{F}\mathbf{K}\mathbf{K}^T - p\mathbf{F}^{-T}, \tag{2.7}$$

$$\mathbf{S} = \mu\mathbf{K}\mathbf{K}^T - p\mathbf{F}^{-1}\mathbf{F}^{-T}. \tag{2.8}$$

These expressions are reduced to that of classical elasticity

$$\mathbf{P} = \mu\mathbf{F} - p\mathbf{F}^{-T},$$

$$\mathbf{S} = \mu \mathbf{I} - p \mathbf{F}^{-1} \mathbf{F}^{-T},$$

when there are no dislocations in the body. The equilibrium equations are in essence the equations for the equilibrium of physical forces when body forces are absent. They are given by the relation

$$\text{Div}\, \mathbf{P} = 0. \tag{2.9}$$

In index notation, the dislocation density tensor is given by the formula

$$\hat{\alpha}_{C\alpha} = \varepsilon_{ABC} \mathbf{K}^{-1}_{\alpha A, B} \; (= \text{Curl}\, \mathbf{K}^{-1}) \tag{2.10}$$

for its two-point expression. The full material dislocation density tensor is

$$\alpha_{DC} = \hat{\alpha}_{D\alpha} \mathbf{K}^{T}_{\alpha C} \; (= \hat{\alpha} \mathbf{K}^{T}), \tag{2.11}$$

since it is a pull-back of $\hat{\alpha}$ to B_R. The second index of α represents the direction of the Burgers vector while the first denotes the direction of the dislocation line. In all the three cases considered in Sect. 3, it holds that $\alpha = \hat{\alpha}$.

3 Continuously Dislocated Bodies Subjected to Anti-plane Shear

In what follows, we consider three particular expressions for the tensor field $\mathbf{K}^{-1}$. These expressions correspond to three different types of continuous distribution of dislocations, one of the screw and two of the edge type. For these forms of the tensor field $\mathbf{K}^{-1}$ we construct the governing field equations of equilibrium. In general, components of this tensor will appear in the field equations as it can be seen from the constitutive expression (2.7). For the first two types of dislocations—of the screw and one of the edge type—we investigate what form should the components of the tensor field $\mathbf{K}^{-1}$ take in order for the material to admit states of anti-plane shear. For the second edge type of dislocations, we provide a particular solution of the field equations under certain conditions.

3.1 Continuous Distribution of Screw Dislocations

A type of $\mathbf{K}^{-1}$ that corresponds to a continuous distribution of screw dislocations is the following

$$\mathbf{K}^{-1}(X_1, X_2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ K^{-1}_{31}(X_2) & K^{-1}_{32}(X_1) & 1 \end{pmatrix}. \tag{3.1}$$

The only non-vanishing term of the dislocation density tensor α is the α_{33} component. It takes the following form

$$\alpha_{33} = K^{-1}_{31,2}(X_2) - K^{-1}_{32,1}(X_1). \tag{3.2}$$

The components K^{-1}_{31}, K^{-1}_{32} of the tensor field $\mathbf{K}^{-1}$ cannot vanish, since this would indicate the absence of dislocations, namely, $\mathbf{K}^{-1} = \mathbf{I}$. Also, another constraint that these quantities should obey is the following

$$\alpha_{33} \neq 0 \quad \text{or} \quad K^{-1}_{31,2}(X_2) \neq K^{-1}_{32,1}(X_1). \tag{3.3}$$

If (3.3) does not hold, the field $\mathbf{K}^{-1}$ can be written as a gradient. Indeed, (3.3) with the equality sign tells us that the derivatives of the components $K_{31}^{-1}(X_2)$, $K_{32}^{-1}(X_1)$ are equal. But, these expressions have different arguments so they will be equal, if they are both equal to the same non-vanishing constant m. Then by integrating, we obtain

$$K_{31}^{-1}(X_2) = mX_2 \quad \text{and} \quad K_{32}^{-1}(X_1) = mX_1, \tag{3.4}$$

after setting equal to zero the constants of integration. So, there exists a scalar field of the form $u = mX_1X_2$, such that

$$K_{31}^{-1}(X_2) = u_{,1} \quad \text{and} \quad K_{32}^{-1}(X_1) = u_{,2}\,. \tag{3.5}$$

Thus, a tensor field $\mathbf{K}^{-1}$ not obeying the constraint of (3.3) can be written in the form of (2.2) and therefore cannot describe dislocations. Also, since the only non-vanishing component of the dislocation density tensor is the α_{33} component, we have a continuous distribution of screw dislocations with Burgers vector and dislocation line parallel to the X_3-axis.

By using (2.7) and (3.1), the first Piola-Kirchhoff stress tensor becomes

$$\begin{aligned}
&P_{11} = \mu - p = P_{22},\\
&P_{21} = 0 = P_{12},\\
&P_{31} = \mu(u_{,1} - K_{31}^{-1}(X_2)),\\
&P_{32} = \mu(u_{,2} - K_{32}^{-1}(X_1)),\\
&P_{33} = \mu\left[-u_{,1}K_{31}^{-1}(X_2) - u_{,2}K_{32}^{-1}(X_1) + (K_{32}^{-1}(X_1))^2 + (K_{31}^{-1}(X_2))^2 + 1\right] - p,\\
&P_{13} = -\mu K_{31}^{-1}(X_2) + pu_{,1}\,,\\
&P_{23} = -\mu K_{32}^{-1}(X_1) + pu_{,2}\,.
\end{aligned} \tag{3.6}$$

The above expressions are reduced to the classical one for elasticity when $\mathbf{K}^{-1} = \mathbf{I}$ (see for instance Knowles [11]).

The equilibrium equations for this case take the form

$$\begin{aligned}
-p_{,1} + p_{,3}u_{,1} &= 0,\\
-p_{,2} + p_{,3}u_{,2} &= 0,\\
\mu\Delta u &= p_{,3}\,.
\end{aligned} \tag{3.7}$$

Therefore, for the case under consideration, the field equations, written in terms of the displacements, are not affected by the dislocations presence. Equations (3.7) have the same form with that of a homogeneous neo-Hookean body [11]. In the same paper it has been examined under which conditions these three equations can be reduced to a single one by finding a relation of the pressure field with the out-of-plane displacement

$$p(X_1, X_2, X_3) = cX_3 + cu(X_1, X_2) + d, \tag{3.8}$$

where d is an arbitrary constant. What remains is the following differential equation

$$\mu\Delta u = c, \tag{3.9}$$

where c is the axial pressure gradient, namely, a constant, present in the linear term giving the dependence of the pressure field on X_3 [10]. The states of anti-plane shear are the ones that obey (3.9).

As Knowles [10] notes, when the traction along a generator of the cylindrical boundary is independent of X_3, then $c = 0$. For this case, the choice of simple shear

$$u(X_1, X_2) = k_1 X_1 + k_2 X_2, \tag{3.10}$$

with k_1, k_2 constants qualifies as a solution.

To conclude, we should remark that, for the case of screw dislocations under consideration, the out-of-plain displacement is not affected at all from the defects presence. Therefore, whatever the value of α_{33} might be, the body will be capable of sustaining anti-plane shear as if the defects were not present, namely, as in the case of a homogeneous neo-Hookean body.

It is worth-noting that the same distribution of screw dislocations together with the same expression for the energy has also been examined by Acharya [1]. There, the purpose was to solve (2.9) and (2.10) for a given dislocation density tensor.

3.2 Continuous Distribution of Edge Dislocations (Case A)

We choose the field $\mathbf{K}^{-1}$ as follows

$$\mathbf{K}^{-1}(X_1) = \begin{pmatrix} 1 & K_{12}^{-1}(X_1) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.11}$$

For this expression the only non-vanishing component of the dislocation density tensor α is the α_{31}. It takes the form

$$\alpha_{31} = -K_{12,1}^{-1}. \tag{3.12}$$

The component K_{12}^{-1} cannot vanish as this would indicate the absence of dislocations, $\mathbf{K}^{-1} = \mathbf{I}$. Also

$$\alpha_{31} \neq 0 \quad \text{or} \quad -K_{12,1}^{-1} \neq 0. \tag{3.13}$$

If the latter constraint does not hold, the tensor $\mathbf{K}^{-1}$ can be written as a gradient. In particular, it corresponds to a simple shear since it can be written as the gradient of the following deformation field

$$\begin{aligned} x_1 &= X_1 + m X_2, \\ x_2 &= X_2, \\ x_3 &= X_3, \end{aligned} \tag{3.14}$$

where m $(= K_{12}^{-1})$ is a constant. Also, the choice of $\mathbf{K}^{-1}$ in (3.11) corresponds to a continuous distribution of edge dislocations with Burgers vector in the X_1 direction and dislocation line in the X_3 direction.

After (2.7) and (3.11), the first Piola-Kirchhoff stress tensor becomes

$$
\begin{aligned}
P_{11} &= \mu[1 + (K_{12}^{-1})^2] - p,\\
P_{21} &= -\mu K_{12}^{-1} = P_{12},\\
P_{31} &= \mu\{u,_1 [1 + (K_{12}^{-1})^2] - u,_2 K_{12}^{-1}\},\\
P_{32} &= \mu(-u,_1 K_{12}^{-1} + u,_2),\\
P_{33} &= \mu - p = P_{22},\\
P_{13} &= pu,_1,\\
P_{23} &= pu,_2.
\end{aligned}
\tag{3.15}
$$

Equations (3.15) provide us with the equilibrium equations in the following form

$$
\begin{aligned}
2\mu K_{12}^{-1} K_{12,1}^{-1} - p,_1 + p,_3 u,_1 &= 0,\\
-\mu K_{12,1}^{-1} - p,_2 + p,_3 u,_2 &= 0,\\
\mu\Delta u + \mu u,_{11} (K_{12}^{-1})^2 + 2\mu u,_1 K_{12}^{-1} K_{12,1}^{-1} - \mu K_{12}^{-1}(u,_{12} + u,_{21}) - \mu u,_2 K_{12,1}^{-1} &= p,_3.
\end{aligned}
\tag{3.16}
$$

The left hand side of the third equation is a function of X_1 and X_2. Therefore, the pressure field should depend at most linearly on X_3. That is

$$
p = f(X_1, X_2)X_3 + p_1(X_1, X_2),
$$

where f is an arbitrary function. But, from $(3.16)_1$ and $(3.16)_2$ we see that $p,_1$ and $p,_2$ are independent of X_3. Thus, for the function f we should have $f,_1 = 0$ and $f,_2 = 0$, that is, $f =$ constant. Therefore, for the pressure field we have

$$
p = cX_3 + p_1(X_1, X_2). \tag{3.17}
$$

The constant c is again the axial pressure gradient.

We insert the above expression of the pressure into (3.16_1) to obtain

$$
2\mu K_{12}^{-1} K_{12,1}^{-1} - p_{1,1} + cu,_1 = 0. \tag{3.18}
$$

By integrating this equation with respect to X_1 we take

$$
p_1 = \mu(K_{12}^{-1})^2 + cu - \omega(X_2), \tag{3.19}
$$

where ω is an arbitrary function. Introducing (3.17) and (3.19) into (3.16_2) we obtain

$$
\mu K_{12,1}^{-1} = \omega,_2. \tag{3.20}
$$

This equation should be satisfied identically if the material is to admit states of anti-plane shear. Namely, if this is the case, then, there exists a pressure field such that the three differential equations of equilibrium can be reduced to a single field equation. This pressure field is given by (3.17) together with (3.19) and the states of anti-plane shear are those u that satisfy (3.16_3), which is the only remaining differential equation of equilibrium.

By inspecting (3.20), one observes that its left hand side is a function of X_1 while its right hand side is a function of X_2. Therefore, in general, (3.20) is not satisfied for an arbitrary choice of the component K_{12}^{-1}. The only choice for this function that makes (3.20) to be satisfied is the following

$$\mu K_{12,1}^{-1} = m \quad \text{and} \quad \omega_{,2} = m, \tag{3.21}$$

where, m is an arbitrary nonzero constant. It should not be equal to zero since in that case K_{12}^{-1} is integrable thus the dislocations field vanishes.

By integrating (3.21) we obtain

$$K_{12}^{-1} = \frac{mX_1}{\mu} \quad \text{and} \quad \omega(X_2) = mX_2. \tag{3.22}$$

Equation (3.22) provides the conditions under which the differential equations of equilibrium can be consistently reduced to a single differential equation for u. In the expression for the component K_{12}^{-1} we assumed that the constant appearing due to the integration is zero. The only non-vanishing component of the dislocation density tensor for the above choice of K_{12}^{-1} is

$$\alpha_{31} = -\frac{m}{\mu}.$$

It corresponds to a homogeneous distribution of dislocations.

To sum up, for $K_{12}^{-1} = mX_1/\mu$ and $\omega(X_2) = mX_2$, where m ($\neq 0$) is an arbitrary constant, there exists a pressure field such that the first two equations of equilibrium hold. This field is given by (3.17), (3.19) and (3.22) and takes the form

$$p = cX_3 + cu + \frac{m^2(X_1)^2}{\mu} - mX_2. \tag{3.23}$$

For this choice of p, the only remaining differential equation of equilibrium that u should satisfy is (3.16$_3$) which can be written as

$$\left[\mu + \frac{m^2(X_1)^2}{\mu}\right]u_{,11} + 2(-mX_1)u_{,12} + \mu u_{,22} + 2\frac{m^2 X_1}{\mu}u_{,1} - mu_{,2} = c. \tag{3.24}$$

It is an elliptic equation since

$$\Delta = B^2 - A\Gamma = (-mX_1)^2 - \left[\mu + \frac{m^2(X_1)^2}{\mu}\right]\mu = -\mu^2 < 0, \tag{3.25}$$

which seems to be a reasonable result since the problem under consideration is within elastostatics.

3.3 Continuous Distribution of Edge Dislocations (Case B)

We examine now the case

$$\mathbf{K}^{-1}(X_3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & K_{32}^{-1}(X_3) & 1 \end{pmatrix}. \tag{3.26}$$

Equation (3.26) corresponds to a continuous distribution of edge dislocations. The Burgers vector is parallel to the X_3 direction and the dislocation line to the X_1 direction, since the only non-vanishing component of the dislocation density tensor is the α_{13} component. The latter takes the form

$$\alpha_{13} = K^{-1}_{32,3}. \tag{3.27}$$

For similar reasons as before

$$\alpha_{13} \neq 0, \tag{3.28}$$

since this would indicate that the tensor field $\mathbf{K}^{-1}$ corresponds to the deformation gradient of an anti-plane shear.

The expression of the first Piola-Kirchhoff stress tensor becomes

$$\begin{aligned}
P_{11} &= \mu - p = P_{22}, \\
P_{21} &= 0 = P_{12}, \\
P_{31} &= \mu u,_1 , \\
P_{32} &= \mu(-u,_2 - K^{-1}_{32}), \\
P_{33} &= \mu[-K^{-1}_{32} u,_2 + (K^{-1}_{32})^2 + 1] - p, \\
P_{13} &= p u,_1 , \\
P_{23} &= -\mu K^{-1}_{32} + p u,_2 .
\end{aligned} \tag{3.29}$$

For this choice of $\mathbf{K}^{-1}$ the differential equations of the equilibrium take the following form

$$\begin{aligned}
-p,_1 + p,_3 u,_1 &= 0, \\
-\mu K^{-1}_{32,3} - p,_2 + p,_3 u,_2 &= 0, \\
\mu \Delta u - \mu K^{-1}_{32,3} u,_2 + 2\mu K^{-1}_{32} K^{-1}_{32,3} &= p,_3 .
\end{aligned} \tag{3.30}$$

We integrate the third equation with respect to X_3

$$p = \mu \Delta u X_3 - \mu K^{-1}_{32} u,_2 + \mu (K^{-1}_{32})^2 + \omega(X_1, X_2), \tag{3.31}$$

where ω is an arbitrary function. We write (3.30_1) and (3.30_2) as follows

$$\begin{aligned}
p,_1 &= p,_3 u,_1 , \\
\mu K^{-1}_{32,3} + p,_2 &= p,_3 u,_2 .
\end{aligned} \tag{3.32}$$

Subcase a

We will examine what happens when $\{u,_1 \neq 0 \text{ and } u,_2 \neq 0\}$. Under this assumption (3.32) can be written

$$\begin{aligned}
\frac{p,_1}{u,_1} &= p,_3 , \\
\frac{\mu K^{-1}_{32,3} + p,_2}{u,_2} &= p,_3 .
\end{aligned} \tag{3.33}$$

By eliminating the terms $p_{,3}$ from the last two differential equations we can write the system of (3.30) in the following form

$$\frac{p_{,1}}{u_{,1}} = \frac{\mu K^{-1}_{32,3} + p_{,2}}{u_{,2}}, \qquad p = \mu \Delta u X_3 - \mu K^{-1}_{32} u_{,2} + \mu (K^{-1}_{32})^2 + \omega(X_1, X_2). \tag{3.34}$$

Inserting (3.34_2) into (3.34_1), we obtain

$$\frac{\mu \frac{\partial}{\partial X_1}(\Delta u) X_3 - \mu K^{-1}_{32} u_{,21} + \omega_{,1}}{u_{,1}} = \frac{\mu \frac{\partial}{\partial X_2}(\Delta u) X_3 - \mu K^{-1}_{32} u_{,22} + \omega_{,2} + \mu K^{-1}_{32,3}}{u_{,2}}. \tag{3.35}$$

The latter can also be written as

$$A(X_1, X_2) \frac{d K^{-1}_{32}}{d X_3} + B(X_1, X_2) K^{-1}_{32} + \Gamma(X_1, X_2, X_3) = 0, \tag{3.36}$$

where

$$\begin{aligned} A(X_1, X_2) &= \mu u_{,1}, \\ B(X_1, X_2) &= -\mu u_{,1} u_{,2} 2 + \mu u_{,2} u_{,21}, \\ \Gamma(X_1, X_2, X_3) &= \mu X_3 \left[u_{,1} \frac{\partial}{\partial X_2}(\Delta u) - u_{,2} \frac{\partial}{\partial X_1}(\Delta u) \right] + (u_{,1} \omega_{,2} - u_{,2} \omega_{,1}). \end{aligned} \tag{3.37}$$

Equation (3.35), or its equivalent form (3.36), is the remaining differential equation that u should satisfy. For an arbitrary expression of K^{-1}_{32} there are no admissible solutions for (3.35) since these solutions will give u as a function of X_3. We go on now to examine under what conditions for K^{-1}_{32} there will exist admissible u from (3.35) and, if possible, to determine a particular solution. For that purpose we view (3.36) as an equation for K^{-1}_{32}. In general, the solutions of the (3.36) will give K^{-1}_{32} as a function of X_1, X_2 and X_3. But, K^{-1}_{32} is a function only of X_3.

Consider the particular case for which there exists a non-vanishing function $\Psi = \Psi(X_1, X_2)$ such that

$$\begin{aligned} A(X_1, X_2) &= \alpha \Psi(X_1, X_2), \\ B(X_1, X_2) &= \beta \Psi(X_1, X_2), \\ \Gamma(X_1, X_2, X_3) &= \gamma \Psi(X_1, X_2) \lambda(X_3), \end{aligned} \tag{3.38}$$

with α, β, γ real constants. Then the differential equation (3.36) takes the form

$$\frac{d K^{-1}_{32}}{d X_3} + \frac{\beta}{\alpha} K^{-1}_{32} + \frac{\gamma}{\alpha} \lambda(X_3) = 0. \tag{3.39}$$

It is a linear ordinary differential equation for K^{-1}_{32} thus giving the admissible expression

$$K^{-1}_{32} = e^{-\int \frac{\beta}{\alpha} d X_3} \left[k - \int \frac{\gamma}{\alpha} \lambda(X_3) e^{\int \frac{\beta}{\alpha} d X_3} d X_3 \right], \tag{3.40}$$

where k is an arbitrary constant. The constant α belongs to the set $R - \{0\}$. The vanishing of this quantity together with (3.37_1) and (3.38_1) leads to $u_{,1} = 0$, which is a non-admissible result. Also, β and γ cannot be equal to zero simultaneously, since this would indicate the absence of dislocations as one can conclude from (3.39). This can be written as $\{\gamma, \beta \in R$ and $\gamma^2 + \beta^2 \neq 0\}$.

Thus, with K_{32}^{-1} given by (3.40) every u that can be written in the form (3.38) is a solution of the equilibrium equations. Then, (3.38) take the form

$$A(X_1, X_2) = \frac{\alpha}{\beta} B(X_1, X_2) = \frac{\alpha}{\gamma} C(X_1, X_2). \tag{3.41}$$

In terms of u we rewrite this equation as

$$u,_1 = \frac{\alpha}{\beta}(-u,_1 u,_2 2 + u,_2 u,_{21}) = \frac{\alpha}{\gamma}\left[u,_1 \frac{\partial}{\partial X_2}(\Delta u) - u,_2 \frac{\partial}{\partial X_1}(\Delta u)\right]. \tag{3.42}$$

To sum up, for a generic expression of K_{32}^{-1} the equilibrium equation does not have admissible solutions. But, for a K_{32}^{-1} given by (3.40) every u that satisfies (3.42) is a solution to our problem. The pressure field will be given by (3.31).

As for the function $\lambda = \lambda(X_3)$, we work as follows. From (3.37_3) we write

$$\Gamma(X_1, X_2, X_3) = X_3 C(X_1, X_2) + D(X_1, X_2), \tag{3.43}$$

where

$$\begin{aligned} C(X_1, X_2) &= \mu u,_1 \frac{\partial}{\partial X_2}(\Delta u) - \mu u,_2 \frac{\partial}{\partial X_1}(\Delta u), \\ D(X_1, X_2) &= u,_1 \omega,_2 - u,_2 \omega,_1 . \end{aligned} \tag{3.44}$$

Γ takes the required form of (3.38) if $\lambda(X_3) = X_3$ and $D(X_1, X_2) = 0$. This means that $u,_1 \omega,_2 - u,_2 \omega,_1 = 0$ and since we have assumed $u,_1 \neq 0$ and $u,_2 \neq 0$, a sufficient selection for ω is to be constant. For $\lambda = X_3$ (3.40) takes the following form

$$K_{32}^{-1} = k e^{-\frac{\beta}{\alpha} X_3} - \frac{\gamma}{\beta} X_3 + \frac{\gamma\alpha}{\beta^2}. \tag{3.45}$$

For $\omega = \text{constant} = d$ the pressure field can be written from (3.31) as follows

$$p = \mu \Delta u X_3 - \mu K_{32}^{-1} u,_2 + \mu (K_{32}^{-1})^2 + d. \tag{3.46}$$

In order to find a particular u that can be written in the form (3.38), namely, a u that satisfies (3.42), we assume that the independent variables of u can be separate in the following additive form

$$u(X_1, X_2) = u_1(X_1) + u_2(X_2). \tag{3.47}$$

In Appendix, it is shown that under the particular case where $\beta = 0$, a function u of the form

$$u(X_1, X_2) = \left\{ \begin{array}{ll} c_2\sqrt{\frac{\alpha}{c}} e^{\sqrt{\frac{c}{\alpha}} X_1} - c_3\sqrt{\frac{\alpha}{c}} e^{-\sqrt{\frac{c}{\alpha}} X_1} + c_6 - \frac{\gamma}{c} X_2 + c_1, & \text{if } c\alpha > 0, \\ \frac{1}{i} c_4\sqrt{-\frac{\alpha}{c}} e^{i\sqrt{-\frac{c}{\alpha}} X_1} - \frac{1}{i} c_5\sqrt{-\frac{\alpha}{c}} e^{-i\sqrt{-\frac{c}{\alpha}} X_1} + c_6 - \frac{\gamma}{c} X_2 + c_1, & \text{if } c\alpha < 0 \end{array} \right\} \tag{3.48}$$

qualifies as a solution to our problem. Indeed, the above u is of the form (3.47). Also, there exists a function Ψ, such that this u can be written in the form of (3.38). For this function we have

$$\Psi = \frac{\mu}{\alpha} u,_1 .$$

To sum up, a function u of the form (3.48) qualifies as a solution to our problem with K_{32}^{-1} taken from (3.39) with $\beta = 0$, i.e.,

$$K_{32}^{-1} = k - \frac{\gamma}{2\alpha}(X_3)^2 \tag{3.49}$$

and the pressure field given by (3.46).

Subcase b

We go on now to examine what happens when $\{u,_1 = 0$ and $u,_2 = 0\}$. In this case u corresponds to a rigid body motion because it is a simple constant for every point of the body. Then, the differential equations for equilibrium, i.e., (3.30) take the following form

$$\begin{aligned} -p,_1 &= 0, \\ -\mu K_{32,3}^{-1} - p,_2 &= 0, \\ 2\mu K_{32}^{-1} K_{32,3}^{-1} &= p,_3 . \end{aligned} \tag{3.50}$$

We directly extract from (3.50_1) that $p = p(X_2, X_3)$. Also, by integrating (3.50_3) with respect to X_3, we obtain

$$p(X_2, X_3) = \mu (K_{32}^{-1})^2 + N(X_2), \tag{3.51}$$

where N is an arbitrary function. Inserting this expression into (3.50_2), we arrive at the following equation

$$-\frac{dN(X_2)}{dX_2} = \mu \frac{dK_{32}^{-1}(X_3)}{dX_3}. \tag{3.52}$$

In the above equation we have the equality of two functions with different arguments. This can be feasible only when both sides of (3.52) are equal to the same constant. Namely, when

$$-\frac{dN(X_2)}{dX_2} = n \quad \text{and} \quad \mu \frac{dK_{32}^{-1}(X_3)}{dX_3} = n, \tag{3.53}$$

with n is an arbitrary non-vanishing constant. By integrating we obtain

$$N(X_2) = -nX_2 - c_1 \quad \text{and} \quad K_{32}^{-1}(X_3) = \frac{n}{\mu} X_3 + \frac{c_2}{\mu}, \tag{3.54}$$

where c_1, c_2 are arbitrary constants.

Therefore, the rigid body motion $u =$ constant qualifies as a solution with pressure field

$$p(X_2, X_3) = \mu \left(\frac{n}{\mu} X_3 + \frac{c_2}{\mu} \right)^2 - nX_2 - c_1 \tag{3.55}$$

and for the component K_{32}^{-1} taking the following expression

$$K_{32}^{-1}(X_3) = \frac{n}{\mu}X_3 + \frac{c_2}{\mu}. \tag{3.56}$$

This expression of K_{32}^{-1} corresponds to a homogeneous distribution of dislocations. The only non-vanishing component of the dislocation density tensor is

$$\alpha_{13} = \frac{n}{\mu}. \tag{3.57}$$

Remark 1 Starting from (2.10) and applying the Curl operator on both of its sides, one will arrive at the following expression

$$\mathrm{Curl}\,\hat{\alpha} = \mathrm{Grad}(\mathrm{Div}\,\mathbf{K}^{-1}) - \mathrm{Div}(\mathrm{Grad}\,\mathbf{K}^{-1}). \tag{3.58}$$

It can be easily seen that for the three special cases considered above, $\mathrm{Div}\,\mathbf{K}^{-1} = 0$. Thus, (3.58) takes the form

$$\mathrm{Curl}\,\hat{\alpha} = -\,\mathrm{Div}(\mathrm{Grad}\,\mathbf{K}^{-1}) \quad \text{or} \quad \Delta\mathbf{K}^{-1} = -\,\mathrm{Curl}\,\hat{\alpha}, \tag{3.59}$$

where Δ denotes the Laplace operator. Therefore, for a given dislocation density tensor $\hat{\alpha}$, (3.59) constitutes a Poisson equation for the components of $\mathbf{K}^{-1}$ that can be solved by the use of the Green's function technique. Substitution of these expressions in the field equations results in a set of equations for the pressure and the out-of-plane displacement. Note that the above derivation is not restricted by the specific expression of the stored energy function. It is mentioned that such a view for (3.59), in a more general setting, has been taken by [4].

Remark 2 The determination of the internal stresses was the main aim of the work of Willis [27]. For the geometrically linear theory, starting from a known dislocation density tensor, one has to solve (2.9) and (2.10) for a zero displacement field. In this case, it also holds that $\hat{\alpha} = \alpha$. To handle equation (2.9), $\mathbf{K}^{-1}$ can be decomposed to a gradient and a non-gradient part [1, 7]. It can be proved that when the defects are distributed homogeneously and the body is not subjected to external loads, the internal stresses vanish [14].

To find out the internal stresses in our analysis, one has to take $\mathbf{F} = \mathbf{I}$ to account for the lack of external loading. For the out-of-plane displacement this results in

$$u_{,1} = 0, \qquad u_{,2} = 0. \tag{3.60}$$

Thus, for the cases where homogeneous distributions of dislocations are considered in this paper one obtains:

(a) For the Case A of edge dislocations, substituting (3.22_1), (3.23) and (3.60) into (3.15), one obtains for the stresses

$$
\begin{aligned}
P_{11} &= \mu\left[1+\left(\frac{mX_1}{\mu}\right)\right] - cX_3 - \frac{m^2(X_1)^2}{\mu} + mX_2,\\
P_{21} &= -mX_1 = P_{12},\\
P_{31} &= 0,\\
P_{32} &= 0,\\
P_{33} &= \mu - cX_3 - \frac{m^2(X_1)^2}{\mu} + mX_2 = P_{22},\\
P_{13} &= 0,\\
P_{23} &= 0.
\end{aligned}
\tag{3.61}
$$

(b) For the Subcase b of Case B of edge dislocations, the admissible solution was found to be the rigid body motion, thus the vanishing of the out-of-plane displacement has been pre-assumed. Equation (3.29) with the help of (3.56) and (3.55) become

$$
\begin{aligned}
P_{11} &= \mu - \frac{n^2}{\mu}X_3 + nX_2 = P_{22},\\
P_{21} &= 0 = P_{12},\\
P_{31} &= 0,\\
P_{32} &= -nX_3,\\
P_{33} &= \mu + n^2\mu(X_3)^2 - \frac{n^2}{\mu}(X_3)^2 + nX_2,\\
P_{13} &= 0,\\
P_{23} &= -nX_3.
\end{aligned}
\tag{3.62}
$$

Equations (3.61) and (3.62) express the internal stresses for the two particular cases considered here. Thus, in contrast to the linear case, homogeneously distributed dislocations do not ensure vanishing of the internal stresses in general.

4 Concluding Remarks

We examined three cases of materially uniform but inhomogeneous bodies subjected to anti-plane shear. More specifically, we have been confined in a particular class of inhomogeneous materials that reduce to the class of Neo-Hookean materials in the homogeneous case. The origin of the inhomogeneity is a fixed distribution of dislocations. For the case of the screw dislocations, it turns out that the field of defects does not affect the out-of-plane displacement. For the edge dislocations of the first case we found the condition that the field of defects should satisfy in order for the equations of equilibrium to be consistent. For the second distribution of edge dislocations we found a particular solution under certain conditions.

If the distribution of dislocations is allowed to change, then plasticity phenomena will take place. The framework proposed here can be extended in this direction by assuming that the tensor field $\mathbf{K}^{-1}$ is not fixed. Thus, the components of $\mathbf{K}^{-1}$ should be treated as unknown functions. In this case, the equations of the equilibrium, proposed in this paper, can not determine the state of the body, i.e., the deformation function $x = x(\mathbf{X})$ and the tensor field of defects $\mathbf{K}^{-1}(\mathbf{X})$. Additional equations for the field of defects are needed for that purpose.

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Appendix

With the assumption of the additive separation of the independent variables of u we have

$$u_{,1} = u_{1,1}, \qquad u_{,11} = u_{1,11}, \qquad u_{,2} = u_{2,2}, \qquad u_{,22} = u_{2,22} \quad \text{and} \quad u_{,12} = u_{1,12} = 0. \tag{A.1}$$

An equivalent form of (3.42) is the following

$$\alpha B = \beta A \quad \text{and} \quad \alpha \Gamma = \gamma A. \tag{A.2}$$

We insert (A.1) into (A.2$_2$) to obtain

$$u_{2,22} = -\frac{\beta}{\alpha}, \tag{A.3}$$

since $u_{,1} = u_{1,1} \neq 0$. With the above expression for u_2 at hand, we are led to

$$\alpha \frac{u_{1,111}}{u_{1,1}} = -\gamma \frac{1}{u_{2,2}} \tag{A.4}$$

with the aid of the second part of (3.42). On the left hand side of (A.4) we have a function of X_1 while on the right we have a function of X_2. Therefore, we conclude that

$$\alpha \frac{u_{1,111}}{u_{1,1}} = c \quad \text{and} \quad -\gamma \frac{1}{u_{2,2}} = c, \tag{A.5}$$

where c is a non-vanishing constant. For (A.5$_2$) to be in agreement with (A.3), we should have $\beta = 0$.

With this proviso for the function u_2 we have

$$u_2 = -\frac{\gamma}{c} X_2 + c_1, \tag{A.6}$$

where c_1 is an arbitrary constant. Then, inserting (A.6) into (A.5$_1$) the following third order ordinary differential equation for the function u_1 remains

$$u_{1,111} = \frac{c}{\alpha} u_{1,1}. \tag{A.7}$$

One can show that the solutions of the above differential equation will be of the form

$$u_1(X_1) = \left\{ \begin{array}{ll} c_2\sqrt{\frac{\alpha}{c}}e^{\sqrt{\frac{c}{\alpha}}X_1} - c_3\sqrt{\frac{\alpha}{c}}e^{-\sqrt{\frac{c}{\alpha}}X_1} + c_6, & \text{if } c\alpha > 0, \\ \frac{1}{i}c_4\sqrt{-\frac{\alpha}{c}}e^{i\sqrt{-\frac{c}{\alpha}}X_1} - \frac{1}{i}c_5\sqrt{-\frac{\alpha}{c}}e^{-i\sqrt{-\frac{c}{\alpha}}X_1} + c_6, & \text{if } c\alpha < 0 \end{array} \right\}, \tag{A.8}$$

where c_2–c_6 are arbitrary constants.

From the expressions (A.6) and (A.8) we see that a function u that satisfies (A.2) takes the following form

$$u(X_1, X_2) = \left\{ \begin{array}{ll} c_2\sqrt{\frac{\alpha}{c}}e^{\sqrt{\frac{c}{\alpha}}X_1} - c_3\sqrt{\frac{\alpha}{c}}e^{-\sqrt{\frac{c}{\alpha}}X_1} + c_6 - \frac{\gamma}{c}X_2 + c_1, & \text{if } c\alpha > 0, \\ \frac{1}{i}c_4\sqrt{-\frac{\alpha}{c}}e^{i\sqrt{-\frac{c}{\alpha}}X_1} - \frac{1}{i}c_5\sqrt{-\frac{\alpha}{c}}e^{-i\sqrt{-\frac{c}{\alpha}}X_1} + c_6 - \frac{\gamma}{c}X_2 + c_1, & \text{if } c\alpha < 0 \end{array} \right\}. \tag{A.9}$$

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